Managing Sponsor Risk in Pension Plans
Dynamic strategies vs. pension assurance

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Rethinking the Economics of Pensions

March 2013, The Royal Statistical Society

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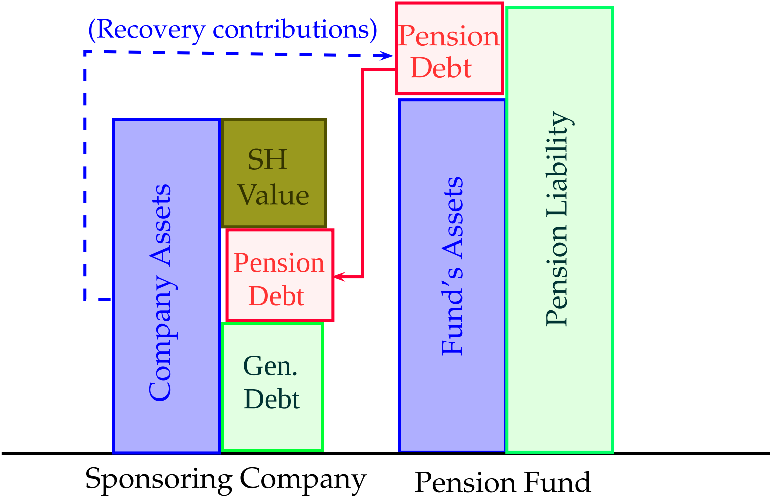
Interconnected balance sheets



Industry analysis: sponsor risk is the elephant in the room

- Failure to manage sponsor risk. Very large historical losses illustrate failure from pension funds to manager sponsor risk, and, in the US, risk-shifting from sponsors.
- Regulations usually ignore contingent asset from sponsor guarantees. Recall that risk-based regulations originate from stand-alone financial entities (banks, then insurance companies).
  - Riskiness of this capital also ignored.
  - Risk-based regulatory framework for pension plans foreshadow a review of the social (employer-employee) pension contract.

Academic literature incomplete on sponsor risk and incentives

- Riskiness of sponsors' guarantees only assessed from the point of view of the public pension insurance company (Sharpe, Treynor, Marcus, Pennacchi and Lewis, Broeders and Chen)
- Risk-shifting (Sharpe, Rauh) not disentangled from risk-management in empirical literature (Rauh)
- Optimal incentive contract (Carpenter) not studied for pension funds and sponsor

Reason for gaps: pensions require merging four strands of literature

- Continuous time finance and portfolio choice
- Laws and regulations
- Corporate finance and incentives
- Insurance

Objective of paper and research questions

- Bridge gaps and contribute simultaneously to the academic literature, industry solution and regulatory thoughts
- Question 1: optimal DB portfolio choice with a sponsor?
  - For DB, hybrid and DC plans
  - Practical hedging challenges
- Question 2: can we build a framework to study the risk-management and risk-shifting nexus in pension plans?
  - Disentangling the two effects
  - Study the form of an incentive contract
- A longer-term view: a basic framework for pension plans opens the door for further studies
  - Regulation on a standalone basis
  - Role of pension plans in overall financial regulations
  - Interaction of pensions with environment (corporate, financial)

Assumptions (see full notations in appendix) :

- Assumption of complete market, with μ_t the risk premium vector, σ_t the volatility of assets and θ_t the risk-premium process.
- Pension fund guaranteed liability L_t with volatility $\sigma_{L\,t}$
- The utility of pensioners is based on wealth relative to promises, *i.e.*, on a funding ratio $F_t = \frac{A_t}{L_t}$. More precisely:

$$U(H_T) = \begin{cases} -\infty & \text{if} \quad H_T < k \\ \frac{{H_T}^{1-\gamma}}{1-\gamma} & \text{if} \quad H_T \geqslant k \end{cases} \tag{1}$$

  where $H_t = F_t + ins_t + cp_t = \tilde{F}_t + ins_t$. The sponsor's recovery contributions $cp_t = min([k - F_T]^, G_T)$ is equal to the deficit $[k - F_T]^+$ but bounded by the sponsor's net asset value G_T.

- The sponsor's share value S is determined endogenously.
- The optimisation is with respect to weights π (not explicit in this presentation)

- Formal derivation via the Martingale approach involves a stochastic discount factor; I use the liability-forward measure $\mathbb{Q}^L$ under which F_t is martingale and budget constraints are not discounted.
- An 'asset-space' simplifies the presentation and the resolution. The solution involves the following steps and building blocks:
  - F^u the optimal funding ratio without a sponsor nor guaranteed income – fixed-mix allocation if (r_t, θ_t) are constant.
  - The optimal prefunded strategy with budget constraint $F_0 + g$ that secures the promise $F_T \geq k$ without a sponsor:

$$F_T^{pref} = \begin{cases} m^* \cdot F_T^u & \text{if} \quad m^* \cdot F_T^u > k \\ k & \text{if} \quad m^* \cdot F_T^u \leqslant k \end{cases}$$

  - The attainment and replication of this optimal payoff when there is a sponsor, *i.e.*, optimally using g the agreed value of the sponsor's contingent recoveries.

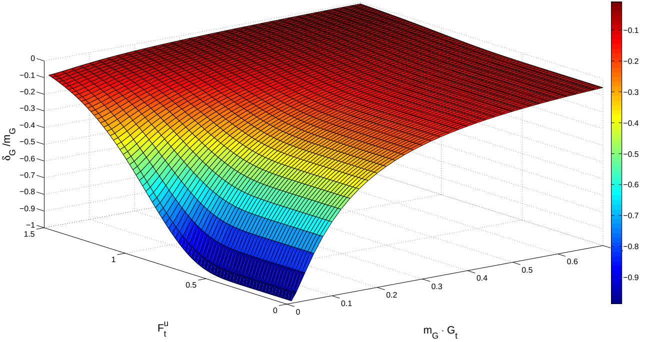
Solution

$$(F_T^*, ins_T^*) = \begin{cases} (F_T^* = F_T^{pref} = m^* F_T^u; ins_T^* = 0) & \text{if} \quad m^* \cdot F_T^u > k \\ \textit{jointly defined } (F_T^* \leqslant k, ins_T^*) & \text{if} \quad m^* \cdot F_T^u \leqslant k \\ \quad\quad \textit{such as} & ins_T^* = k - F_T^* - m_G \cdot G_T \\ \quad\quad \textit{and if sponsor pays ins} & \mathbb{E}_0^{\mathbb{Q}_L}[k - F_T^*]^+ = g \end{cases}$$

Proof and easiness of representation rely on:

- To show that the maximum utility can be attained and find in closed form the payoff and portfolio strategy, we disentangle the 'insurance' component.
- *ins* has no pay-off in the upside, so on the upside $H_T \equiv F_T^{pref}$
- in the downside, $(\tilde{F}_T, F_T) < F_T^{pref}$ to collect recovery contributions
- F and *ins* jointly defined as a function of (G, F^u) to respect both the pension promise or the budget constraint (or *ins* on top of F)

ins is an (additional) *bivariate* 'risk management' component linked to the joint probability of sponsor bankruptcy and PF underfunding.



The position in assets that correlate with the sponsor's equity turns short during stress. By contrast, without a sponsor, 'traditional' portfolio insurance has a *univariate* driver – the risk budget is a measure of a distance to a floor.

Impact of buying insurance against the sponsor's riskiness?

Radically simplifying pricing of claims, we have at $\mathbf{t=0}$:

The *pension fund* has

- $A_0 = 80$ and $L_0 = 100$
- Deficit = 80-20= **20**

The *sponsor* has

- Assets **25**, pension debt **20**
- Share value $\underline{\mathbf{S}}$: $\mathbf{25} - \mathbf{20} = \underline{\mathbf{5}}$

The fund enters a short (OTC) position on the sponsor's asset value.
At $\mathbf{t=1}$, the sponsor's asset value falls by 6 to **19**. Then:

- The pension fund's *ins* contract value rises from 0 to 6
- If A and L unchanged, plan deficit reduced by 6 from 20 to **14**
- The share value $\underline{\mathbf{S}}$ is unchanged at $\mathbf{25} - \mathbf{14} = \underline{\mathbf{5}} \Rightarrow \frac{\Delta S}{\Delta G} \rightarrow 0$
- Benefits: pension debt never triggers the sponsor's bankruptcy
- The issue: the fund (or the OTC contract provider) cannot hedge the change in the sponsor's asset value with the stock.

Implementation issues for quasi 'self-hedges'

- The share value reads: $S_t = G_t - \mathbb{E}_t^{\mathbb{Q}_L}[k - F_T] + ins_t$.
- Remind that $ins \to k - F^* - m_G \cdot G$ and offsets changes in G.
- When $G \downarrow 0$, $\frac{\partial S}{\partial G} \downarrow (1 - m_G)$ and $\frac{\partial ins}{\partial G} \downarrow -\frac{m_G}{1-m_G}$
- Implications: constraints to the hedging scheme:
  - Maximum m_G and hedge ratio
  - Liquidity, leverage constraints, ability to short
  - Third party insurance as solving implementation issues – amongst others, the 'marked to market convexity'.

The Fund's Optimisation Problem under Risk-Shifting.

N.B.: Sponsor's risk-shifting incentives are better understood and solved by a perpetual insurance contract with risk-based pricing, so we focus on pension risk-shifting.

- The fund maximises its expected utility accounting for sponsor's legal contributions but does not aim to control the value of – rather to maximise – recovery contributions.
- With the same utility function Eq (1), the problem reads:

$$\max_{\pi} \mathbb{E}_0[U(H_T)] = \max_{\pi} \mathbb{E}_0[U(\tilde{F}_T + cp_T)]$$
$$\text{s.t. } \mathbb{E}_0^{\mathbb{Q}_L}(\tilde{F}_T) = F_0 \text{ `Asset-only budget constraint'}$$

- Only the 'Asset-only budget constraint' is present: $cp_T = min\left([k - \tilde{F}_T]^+, G\right)$

Solution: Maximum Pension Risk Shifting theorem

- The fund's optimal terminal wealth $\tilde{F}_T^{rs} = F_T^{rs} + ins_T^{rs}$ decomposes as:

$$F_T^{rs} = \begin{cases} m^{rs} \cdot F_T^u & \text{if} \quad m^{rs} \cdot F_T^u > F_T^+ \\ 0 & \text{if} \quad m^{rs} \cdot F_T^u \leqslant F_T^+ \end{cases} \tag{2}$$

and

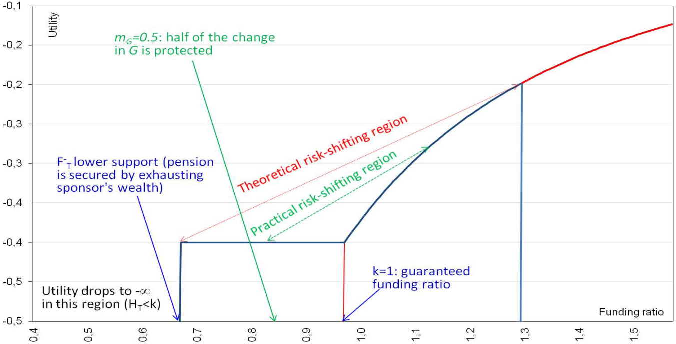
$$ins_T^{rs} = \begin{cases} 0 & \text{if} \quad m^{rs} \cdot F_T^u > F_T^+ \\ [k - G_T]^+ & \text{if} \quad m^{rs} \cdot F_T^u \leqslant F_T^+ \end{cases} \tag{3}$$

where the line $\left([k - G_T]^+, F_T^+\right)$ is tangential to the utility at F_T^+.

- A non-contractible fund targets $m_G = 1$ to maximise wealth extraction. This maximises the probability of being overfunded and also maximises the multiplier m^{rs} (it maximises 'leverage').
- It combines two all-or-nothing quantities, maximising both the upside funding ratio and the downside recoveries.

Figure: Utility Loss from Risk-shifting *w/wo* Asymmetry

Risk-shifting <≃> non-convexity <≃> betting the house. Under asymmetry of information, the limited ability to hedge ($m_G < 1$) shrinks the non-convex region (green dotted curve) relative to $m_G = 1$ (red dotted curve).



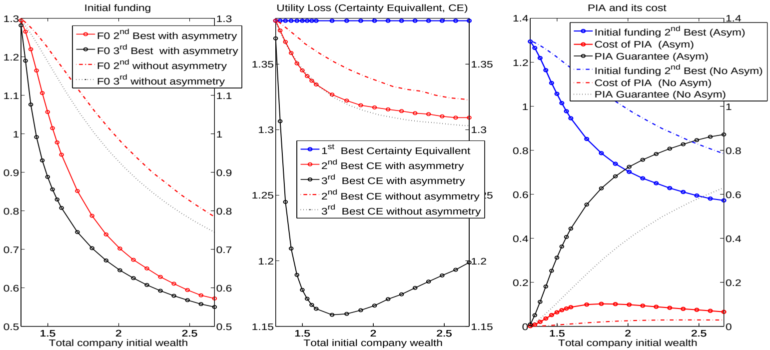
Resolution of risk-shifting incentives: Provide pension assurance (PA), an insurance protection against the sponsor's default

- Limited in amount: c is the maximum payoff.
- Provided that the pension fund behaves well, *i.e.*, does not increase leverage with *ins*, *i.e.*, shorting the sponsor's equity
- c must be sufficient for the fund to accept. . . and behave.
- Then the fund pursues a binary strategy

$$\tilde{F}_T^c = F_T^c = \begin{cases} m^c \cdot F_T^u & \text{if} \quad m^c \cdot F_T^u > F_T^+(c) \\ [k - c] & \text{if} \quad m^c \cdot F_T^u \leqslant F_T^+(c) \end{cases} \qquad (4)$$

- The optimal PA is that accepted which minimises non-convexity, thus with the smallest c.
- PA diminishes non-convexity, the reliance on the sponsor's balance sheet, it increases initial funding, diminishes asymmetry of information and avoids implementation issues.

Figure: PA is an optimal contract for PFs



PFs receive the entire value of funds thus their contract differ from asset managers. Asymmetry (thick curves) reduces initial funding and makes CE lesser than (assumed) cost of pension (annuity worth 1.3). PA reduces significantly costs, improves upon third best without asymmetry. Benefits of PA disappear for very large sponsors.

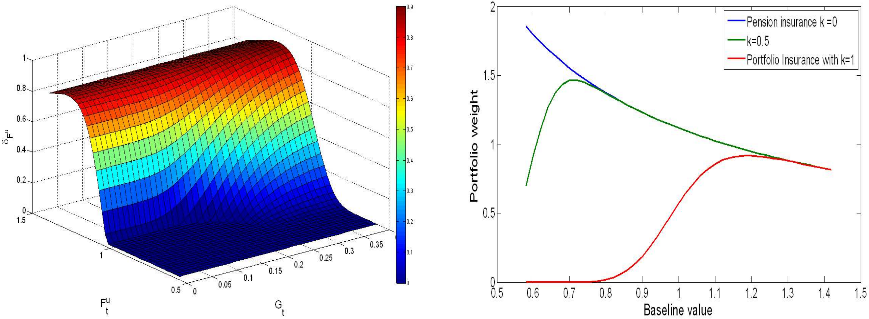
- The regulatory objective of protection of pension rights can best be achieved by a non-redundant contract
  - Pension Assurance (PA) is an implementation device
  - It solves for incentives
- Straightforward extensions for:
  - Corporate finance – plan design for incentives
  - Empirical research on asset allocation decisions
  - Pension regulations
- Solvency II by relying solely on funding:
  - Eliminates the value of contingent sponsor recovery
  - Implicitly transforms DBs into DCs with guarantees
  - Leads to its own implementation issues
  - Transforms the financing ability of firms

- Pension assurance (PA) as an early inspiration for regulations.
- Purely financial techniques (portfolio insurance, PI) gained the primacy since the 90s.
- PI requires liquid assets and is pro-cyclical (Borio).
- PA can foster the regulatory objective of security without compromising either long-term investing or financial stability (as opposed to procyclicality)
- Long-term investing as a buy-and-hold type strategy in illiquid assets. German booked-reserved, PSaVG insured DB plans were fully invested in illiquid assets.

- Financial stability: a formal extension is needed.
- A formal joint study of insurance techniques (PA) and financial ones (PI) would show the aggregate benefits of insurance.
- Pensions can foster financial stability via other tools.
- An *illustration* can be found with the state-dependent supply/demand for risky assets from pension plans – loosely speaking their contribution to financial stability.

Figure: PI vs. PA, external pro-cyclicality, an *illustration*

Shows the pension fund allocation to the unconstrained strategy (w/o insurance) in an ad-hoc specification: same budget constraint, mean-reverting returns, Sharpe-ratio has 90% correlation to changes in stock-price, no (additional) inter-temporal hedging.



PA diminishes pro-cyclicity. Unconstrained, fully counter-cyclical strategy (blue line) only can be achieved with (full) insurance

- P vector of traded risky assets: $dP_t = diag(P_t) \cdot (\mu_t \cdot dt + \sigma_t \cdot dW)$ with μ_t the risk premium vector and σ_t the volatility of assets
- The pension fund guaranteed liability L_t has volatility σ_{Lt}.
- Dynamic market completeness: stochastic discount factor, Z with diffusion $dZ_t = -Z_t \cdot (\theta_t' \cdot dW + r_t \cdot dt)$
- Complete markets with $\mathbb{Q}$ martingale measure $\zeta_t = Z_t e^{\int_0^t (r_t \cdot dt)}$ *w.r.t.* historical probability measure $\mathbb{P}$. $\mathbb{Q}$ has Radon-Nikodym density $\zeta_t = \exp(-\int_0^t \theta_t' dW - \frac{1}{2} \|\theta_t\|^2 dt)$.
- $F_t = \frac{A_t}{L_t}$, the funding ratio at time $0 \leqslant t \leqslant T$, is a martingale under $\mathbb{Q}_L$. A budget constraint on A_T is as a budget constraint on F_T under $\mathbb{Q}_L$, without discounting.

- The sponsor has a net asset value G in the numeraire of the liability, with volatility σ_G. Its share value S is determined endogenously.
- $cp_T = [[k - F_T]^+ \vee G]$ the sponsor's recovery contributions
- m a positive number made to satisfy the budget constraint, interpreted as a multiplier
- m_G a positive number interpreted as a hedge ratio
- g the agreed value of the guarantee
- k the fraction of L_T that trustees must secure (usually $k = 1$).
- ins_T a conceptually separate insurance component against the sponsor's riskiness.
- Utility to be maximised for pension participants:

$$U(H_T) = \begin{cases} -\infty & \text{if} \quad H_T < k \\ \frac{{H_T}^{1-\gamma}}{1-\gamma} & \text{if} \quad H_T \geqslant k \end{cases}$$

The optimal pre-funded strategy F_T^{pref} with initial funding ratio $F_0 + g = H_0$ and no further support from the sponsor is uniquely defined as $[m^* \cdot F_T^u \vee k]$ where $F_T^u = (\upsilon_0 \cdot Z_T \cdot L_T)^{-\frac{1}{\gamma}}$ is the optimal strategy for the unconstrained investor (υ_0, a positive number, ensures the respect of the budget constraint):

- We first solve the **unconstrained** static maximisation program:
  $\max_{\pi} E_0\left[\frac{(A_T/L_T)^{1-\gamma}}{1-\gamma}\right] \quad s.t.\ A_0 = \mathbb{E}_0^{\mathbb{Q}}[\frac{A_T}{e^{rT}}] = \mathbb{E}_0[Z_T \cdot A_T]$
  Given the states of the world, the first-order condition reads
  $\frac{1}{L_T}(A_T^u/L_T)^{-\gamma} = \upsilon_0 \cdot Z_T.$ So, $A_T^u = \upsilon_0{}^{-1/\gamma} \cdot Z_T{}^{-\frac{1}{\gamma}} \cdot L_T{}^{1-\frac{1}{\gamma}}$, and $F_T^u = \upsilon_0{}^{-\frac{1}{\gamma}} \cdot (L_T Z_T)^{-\frac{1}{\gamma}}$
- We then solve for the **constrained** program:
  $\max_{\pi} \mathbb{E}_0\left[\frac{(A/L)^{1-\gamma}}{1-\gamma}\right] \quad \text{s.t. } A_0 = \mathbb{E}_0^{\mathbb{Q}}[Z_T \cdot A_T] \quad \text{and } F_T \geqslant k$
  The FOC read: $\frac{1}{L_T} \cdot F_T^{pref\,-\gamma} - \upsilon_k \cdot Z_T + \frac{\upsilon_c(k)}{L_T} = 0$ where υ_0 and υ_k are the Lagrange multipliers associated with the budget constraint (for $k = 0$ or k), $\upsilon_c(k)$ with the funding constraint k.

- $\upsilon_c(k) = 0$ when $F_T^{pref} > k$, and is otherwise binding, so we have:

$$F_T^{pref} = \begin{cases} \upsilon_k^{-\frac{1}{\gamma}} \cdot (Z_T \cdot L_T)^{-\frac{1}{\gamma}} & \text{if} \quad Z_T \cdot L_T < \underline{ZL} = \frac{k^{-\gamma}}{\upsilon_k} \\ k & \text{if} \quad \underline{ZL} < Z_T \cdot L_T \end{cases} \tag{5}$$

Or:

$$F_T^{pref} = \begin{cases} m^* \cdot F_T^u & \text{if} \quad m^* \cdot F_T^u > k \\ k & \text{if} \quad m^* \cdot F_T^u \leqslant k \end{cases} \tag{6}$$

$\underline{ZL}$ represents the threshold $m^* \cdot F_T^u = k$ and $m = \left(\frac{\upsilon_k}{\upsilon_0}\right)^{-\frac{1}{\gamma}}$.

- Finally, the participation rate m must respect the pre-funded strategy budget constraint $F_0 + g$.

Log-normal case:
When the parameters set is constant, assets, liabilities and the *s.d.f.* are log-normals, and so is $Z^{-\frac{1}{\gamma}}$ and its product by L_T, F^u_T. Noting that $\mathbb{E}^{\mathbb{Q}_L}[k \vee m^* \cdot F^u_T] = k + \mathbb{E}^{\mathbb{Q}_L}[F^u_T - k]^+$ and making use of the Black-Scholes formula implies:

$$F_0 + g = m^* \cdot F_0 \cdot \mathbb{N}(d_+(m^* \cdot F_0, k)) + k \cdot \mathbb{N}(-d_-(m^* \cdot F_0, k)) \quad (7)$$

with $d\pm = \frac{log(m^* \cdot F_0/k) \pm \frac{1}{2}\sigma^2_{F^u} T)}{\sigma_{F^u}\sqrt{T}}$ and $\sigma_{F^u} = \frac{||\theta' - \sigma_L||}{\gamma}$. The volatility of F^u is the difference between σ_{A^u} and σ_L; A^u has exposure $\frac{1}{\gamma}$ to a portfolio with volatility θ' and $1 - \frac{1}{\gamma}$ to a portfolio with volatility that of liabilities.
A more general and elegant proof involves when under $\mathbb{Q}_L$ using F^u as a (relative) numeraire. Then, the budget constraint for m reads: $\mathbb{E}^{\mathbb{Q}_L}[k \vee m^* \cdot F^u_T] = m^* \cdot F_0 \cdot \mathbb{E}^{\mathbb{Q}_A}[\mathbb{1}_{F^u_T > k/m}] + k \cdot \mathbb{E}^{\mathbb{Q}_L}[\mathbb{1}_{F^u_T < k/m}]$.