Sonar equation and the Lambert W function

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16 April 2007

In sonar, signals from a source are degraded by both spherical spreading loss and a frequency-dependent attenuation. If the range from the source is R, the loss is given by the formula

$$L = 20\log(R) + \alpha R \tag{1}$$

Suppose that we are given the loss L and the attenuation coefficient α. We wish to find the range R at which this loss is achieved. A practical application of this is to determine the range over which a signal of a particular source level may be detected by a sonar.

The Lambert W function, $W(x)$, is determined by the formula

$$W(x)\exp(W(x)) = x \tag{2}$$

(http://en.wikipedia.org/wiki/Lambert's_W_function). We can use this function to solve equation (1).

Anti-logging equation (1) leads to the formula,

$$R\,10^{\alpha R/20} = 10^{L/20}$$

This can be rewritten as

$$R\exp(\alpha R\log_e(10)/20) = 10^{L/20}$$

Let

$$\beta = \alpha\log_e(10)/20 \tag{3}$$

Then

$$R\exp(\beta R) = 10^{L/20}$$

Multiplying by β, one gets

$$\beta R\exp(\beta R) = \beta\,10^{L/20}$$

We conclude from equation (2) that

$$R = (1/\beta)\,W(\beta\,10^{L/20}) \tag{4}$$

A fast Matlab m-file is available for computing W, see

http://www.mathworks.com/matlabcentral/fileexchange/loadFile.do?objectId=6909&objectType=FILE

This solution is a lot more efficient than the usual Newton-Raphson method.